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Theorems for Infinite Series $\sum a_n$

1. If $\lim_{n\to\infty} a_n \neq 0$ then the series diverges
2. Ratio test: letting

   $$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = L\,,$$

   then

   (a) if $L < 1$ then the series converges,

   (b) if $L > 1$ then the series diverges,

   (c) if $L = 1$ then not sure.

3. Limit comparison test: Given $\sum a_n$ and $\sum b_n$, with a_n and b_n positive, assume that

   $$\lim_{n\to\infty} \frac{a_n}{b_n} = L\,,$$

   where $0 < L < \infty$. In this case, if $\sum b_n$ converges then so does $\sum a_n$, and if $\sum b_n$ diverges then so does $\sum a_n$.

4. Alternating series test: if the series is alternating and

   (a) $\lim_{n\to\infty} a_n = 0$,

   (b) $|a_{n+1}| \leq |a_n| \;\; \forall n$,

   then the series converges.

Known Series

1. Geometric series $\sum_{n=0}^{\infty} r^n$

   (a) If $-1 < r < 1$ then the series converges and

   $$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}\,.$$

   (b) If $|r| \geq 1$ then the series diverges.

2. $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$.

Power and Taylor Series

1. Taylor polynomial and remainder: $f(x) = p_n(x) + R_n$

$$p_n = f(a) + f'(a)(x-a) + \frac{1}{2}f''(a)(x-a)^2 + \cdots + \frac{1}{n!}f^{(n)}(a)(x-a)^n$$

$$R_n = \frac{1}{(n+1)!}f^{(n+1)}(c)(x-a)^{n+1}$$

2. Given $\sum c_k(x-a)^k$, suppose that

$$\lim_{k\to\infty}\left|\frac{c_k}{c_{k+1}}\right| = R\,,$$

then

   (a) if $R = 0$ then the series only converges when $x = a$,

   (b) if $0 < R < \infty$ then the series converges for $|x-a| < R$ and diverges for $|x-a| > R$,

   (c) if $R = \infty$ then the series converges for all x.

Caveat: The information on these two pages is bare-bones, and more complete versions can be found in the text and/or the class notes.